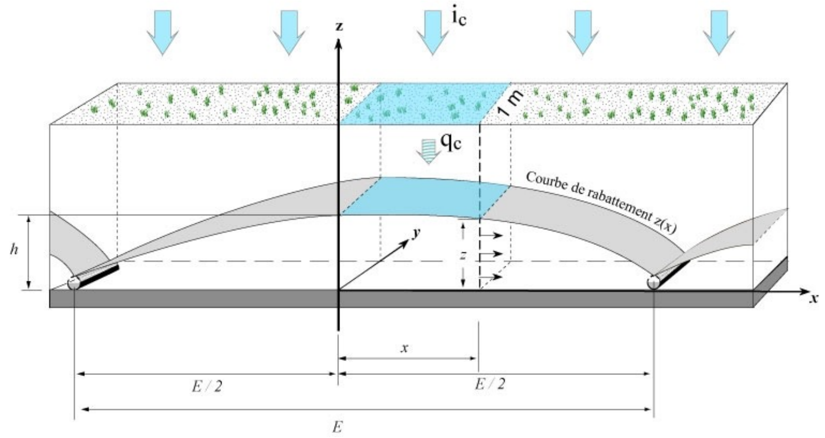
Irrigation and Drainage Engineering

(Soil Water Regime Management)

(ENV-549, A.Y. 2025-26)

4ETCS, Master option

Prof. Paolo Perona
Platform of Hydraulic Constructions



Lecture 10-2. Drainage of agricultural land: design of drains in permanent and variable regimes

Calculation of the drain spacing

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

Dupuit-Forchheimer hypothesis

- homogeneous, isotropic medium
- drains laid on impermeable bedrock
- negligible vertical component of velocity
- identical velocities at all points along a vertical line

Under the assumptions adopted, the flow at each point is therefore proportional to the slope of the water table.

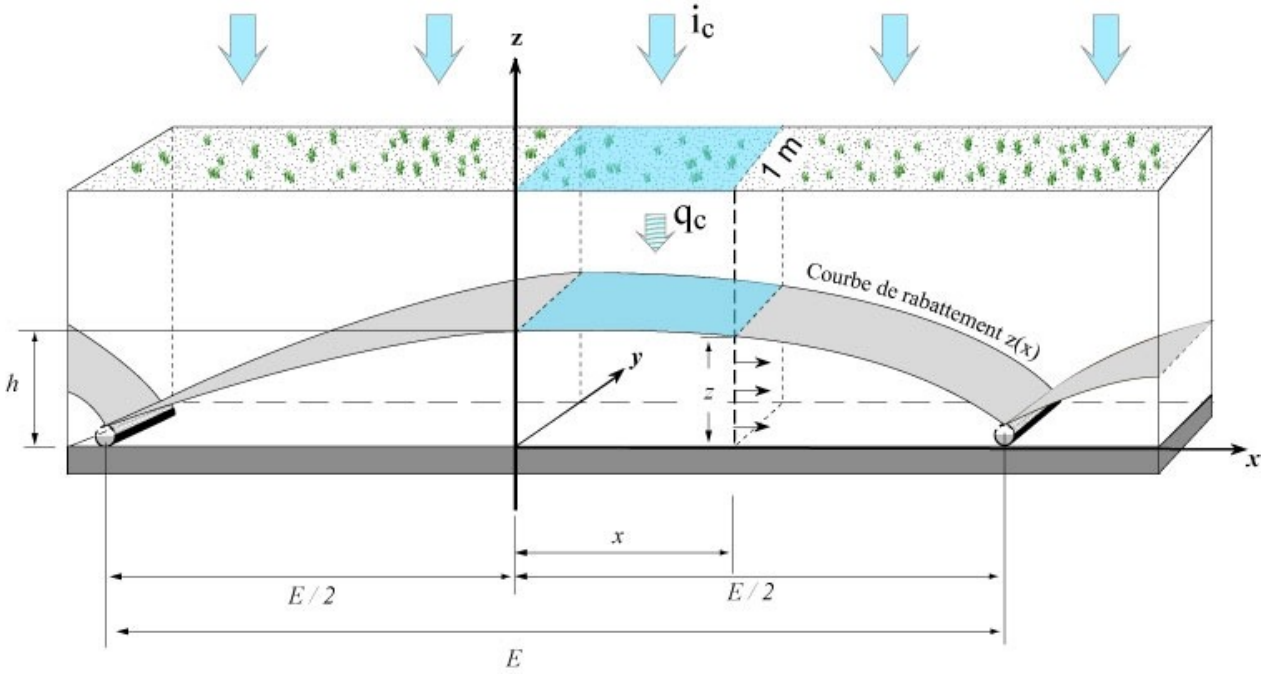
➔ Theoretical vertical equipotential curves

➔ One-dimensional horizontal flow (1D - H) for which the flow is (Darcy) :

$$q = -K\frac{dH}{dx} = -K\frac{dz}{dx}$$

q : flux, i.e. flow per unit area
K : hydraulic conductivity at saturation
H : Hydraulic head
z(x) : depression curve (equation) for the water table

Permanent regime: drains on impermeable layer



$$Q_x = q_c\,x \quad \text{and} \quad Q_x = -K\frac{dz}{dx}z$$

$$q_c\int_0^x x\,dx = -K\int_h^z z\,dz \qquad q_c\frac{x^2}{2} = -\frac{K}{2}\left(z^2 - h^2\right)$$

being: $$q_c\,x^2 = Kh^2 - Kz^2$$

Calculation of the space between drains, E

$x = E/2 \qquad z = 0 \qquad$ being : $q_c\dfrac{E^2}{4} = Kh^2$

And so: $$E = 2h\sqrt{\frac{K}{q_c}}$$

Water table profile equation

$$q_c\,x^2 = Kh^2 - Kz^2$$

$\dfrac{q_c\,x^2}{Kh^2} = 1 - \dfrac{z^2}{h^2} \qquad$ being : $\dfrac{q_c\,x^2}{q_c\dfrac{E^2}{4}} + \dfrac{z^2}{h^2} = 1$

And so*: $$\frac{4x^2}{E^2} + \frac{z^2}{h^2} = 1$$

* Equation of an ellipse with semi-major axis E/2 and semi-small axis h

➔ Cylindrical surface (generatrices parallel to the drain) with an elliptical shape

Space E between drains

$$E = 2h\sqrt{\frac{K}{q_c}}$$

h : height of the water table at the inter-drains

h = p - n (p: depth of drains;

n: minimum depth of water table)

qc: characteristic drainage flow rate

K: hydraulic cond. at soil saturation

Equation of the water table profile

$$\frac{4x^2}{E^2} + \frac{z^2}{h^2} = 1$$

Cylindrical surface of elliptical shape

Discharge q_y per m of drain length

$$q_y = E\, q_c$$

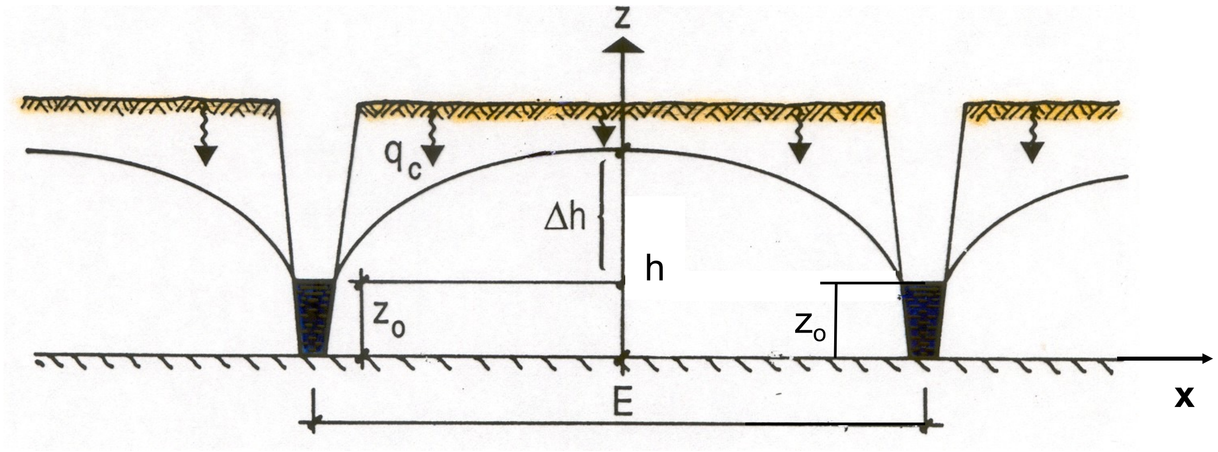
If the area around the drains is not sufficiently filtered, the condition at the drains becomes :

$x = E/2$ **$z = h_e$**

for which the space E between the drains becomes:

$$E = 2\sqrt{\frac{K}{q_c}\left(h^2 - h_e^2\right)}$$

Ditches on impermeable layer, in permanent recharge regime



$$Q_x = q_c\,x \qquad \text{and:} \qquad Q_x = -K\frac{dz}{dx}z$$

$$q_c\int_0^x x\,dx = -K\int_h^z z\,dz \qquad q_c\frac{x^2}{2} = -\frac{K}{2}\left(z^2 - h^2\right)$$

be:

$$q_c\,x^2 = K h^2 - K z^2$$

Calculation of the spacing E

$x = E/2 \qquad z = z_0 \qquad$ be : $\qquad q_c\frac{E^2}{4} = K h^2 - K z_0^2$

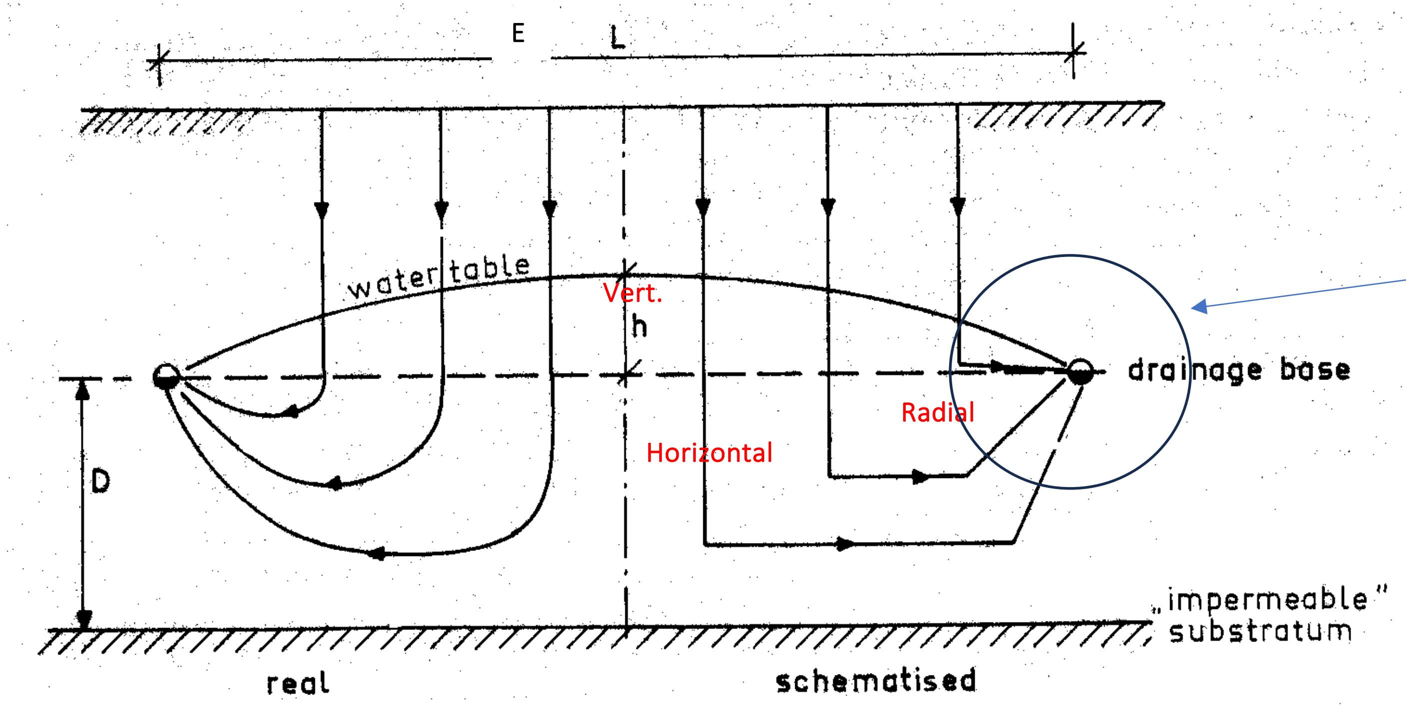
then:

$$E = 2\sqrt{\frac{K}{q_c}\left(h^2 - z_0^2\right)}$$

Or, in other terms:

$$E^2 = \frac{8\,K\,z_o\,\Delta h}{q_c} + \frac{4\,K\,\Delta h^2}{q_c}$$

Case of drains not lying on an impermeable layer



Zone d'écoulement radial : cercle de rayon de l'ordre de 0.7 D, centré sur le drain

Actual and schematic flow towards drains not lying on a impermeable substratum

E >> h et D : essentially horizontal flow

E ≅ D : the radial component can be significant

h >> E : non-negligible vertical component

Analogie de Hooghoudt

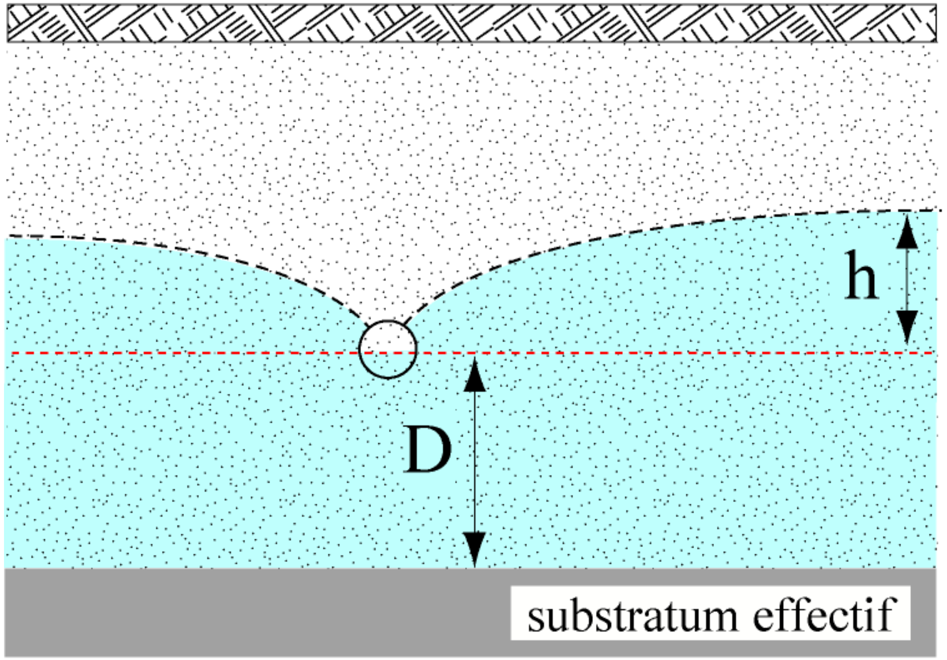
PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

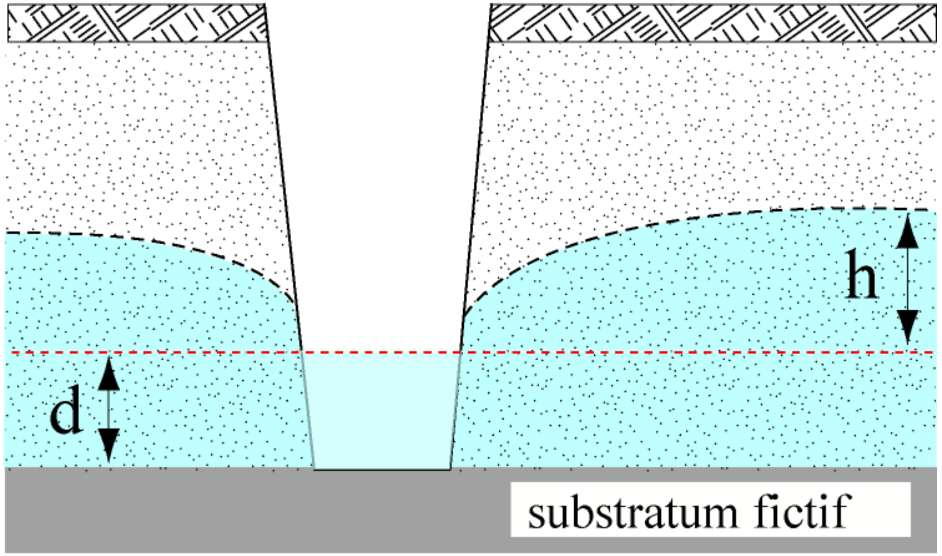
To avoid having to take into account the radial component of the flow towards a drain not resting on the impermeable substratum, Hooghoudt (1940) proposed replacing the network of drains by a supposedly equivalent network of ditches resting on a fictitious impermeable substratum such that, in the 2 cases :

- the discharge rate is the same
- the maximum hydraulic head h is the same
- the distance E between the structures is the same

The fictitious bedrock lies at a depth d below the plane of the drains, known as the equivalent depth..

The advantage of Hooghoudt's analogy is that it replaces the complex real case with an equivalent case whose solution is known, i.e. being the case of a ditch resting directly on the impermeable bedrock.





d < D : profondeur équivalente

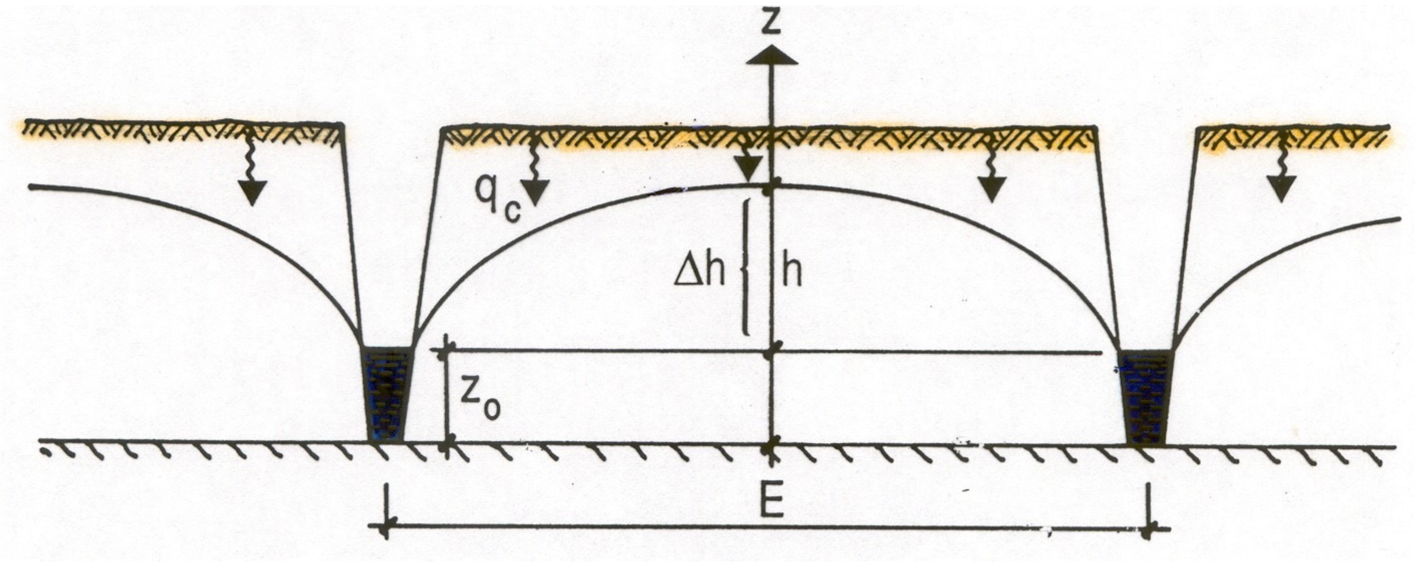
Actual situation

(flow towards a drain not resting on bedrock)

Hooghoudt equivalence

(flow towards a ditch reaching a fictitious impermeable layer)

Parallel ditches on impermeable layer in permanent regime (Hooghoudt analogy)



$$E^2 = \frac{8\,K\,z_o\,\Delta h}{q_c} + \frac{4\,K\,\Delta h^2}{q_c}$$

Hooghoudt analogy: $\Delta h = h$, $z_o = d$

$$E^2 = \frac{8\,K\,h\,d}{q_c} + \frac{4\,K\,h^2}{q_c}$$

Hooghoudt analogy: further comments

1. Possibility of taking account of heterogeneity linked to the presence of different horizons located on either side of the drain planes*:

$$q_c = \frac{8\,K_i\,h\,d}{E^2} + \frac{4\,K_s\,h^2}{E^2}$$

2. When the contribution of the flow above the plane of the drains can be neglected, the Hooghoudt equation reduces to :

$$q = \frac{8\,K\,h\,d}{E^2}$$

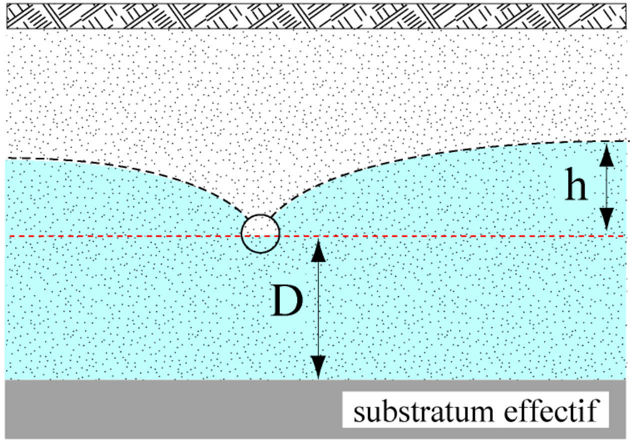
(Simplified Hooghoudt formula)

3. For the same spacing E and the same load h at the inter-drains, the flow rate of the drains is always lower when they rest on an impermeable layer.

$*K_i$: hydraulic conductivity at saturation of the layer beneath the drains

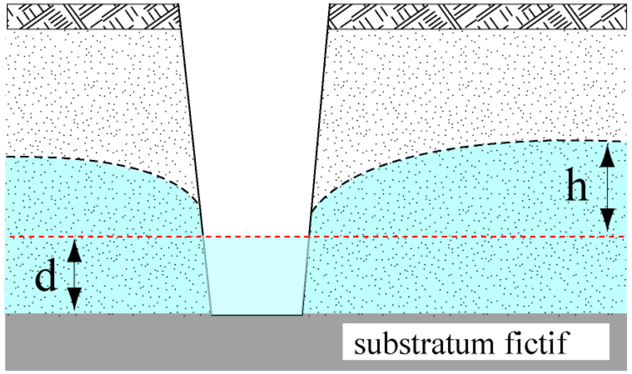
K_s : hydraulic conductivity at saturation of the layer located in the plane of the drains

Houghoudt formula: Calculation of the equivalent depth d



$$d = \frac{D}{\frac{8D}{\pi E}\ln\frac{D}{u} + 1}$$ pour D < E/4

$$d = \frac{\pi E}{8 \ln\frac{E}{u}}$$ pour D > E/4



D: actual depth of the impermeable horizon below the drains

E: drain spacing

u: wetted perimeter of drains (radius r, half full); in general, $u = \pi r$